What is Mathematics? The problem of recontextualisation

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What is School Language?

We are all very familiar with Piaget's (1953) clinical interviews with young children. In one sent of interviews, Piaget poured lemonade from a narrow glass into a wide one and asked, 'have you now got more or less to drink?' When the child replied, say, 'less', Piaget would ask why and the child might say, 'because the level has gone down.' Another child might say 'it's the same.' Piaget again asks, 'why?' and now there are two possible categories of answer: 'because it's shorter, but wider,' or 'because you didn't spill any. Piaget argued that the last answer—which could equally have been, 'I've got less because you did spill some'—was predicated on the conservation of quantity. Equally, the interviewer might pour the lemonade into two glasses and the mature response would be, 'I still have the same amount, because none of it has been spilled.' Oddly, perhaps, when Helen Watson (1987) posed similar problems to monolingual; Yoruba speakers, she found a different answer. Pouring peanuts from one bowl into two bowls, the mature answer was, 'I've got more, because it was one and now it's two.'

Watson used Wittgenstein's idea of language games to explain what was going on: this is the way that Yoruba connects and uses these words, which is different from the way in which English speakers connect and use them. As a sociologist, I'm inclined to speculate on the way in which this Yoruba-speaking culture constructs property; what was only one item of property is now two; but this is mere speculation. What would certainly seem to be the case is that the two different linguistic cultures think of conservation in distinctly different ways. Piaget himself might well reply that the Yoruba culture is authoritative, gerontocratic (I have no evidence to confirm that this either is or is not the case) and that this has resulted in the arresting of the development of operational thought. I'll come back to the issue of authority later, but, in any event, there's clearly something going on that has produced different kinds of 'mathematical' concepts in these two languages.

Alan Bishop (1988a, 1988b) has proposed that mathematics is a pan-human phenomenon and identifies six 'fundamental activities' that are universal and that are both necessary and sufficient for the development of mathematics, they are: counting, locating, measuring, designing, playing, and explaining. Let's consider one of these, 'locating'. Pam Harris (1991) has pointed out that the Aboriginal people with whom she worked made far more use of the points of the compass than left and right when orienting themselves in space. Harris was told, for example, about a woman who, when giving birth in a hospital, complained of a pain in her east leg. Now I tend to have some difficulty in distinguishing left from right; I'm told that this is not uncommon in people who, like me, tend to prefer their left side. But I have to think far harder to distinguish the points of the compass in relation to my own position. Sitting here in my apartment, looking out over the dock and the River Thames, I think my left hand is on my north-ish side, but I had to think quite hard with references to recalled maps of London and the direction that the river goes just here. Incidentally, I have no problem in distinguishing left from right in Japanese. I can only think that this is because I still remember where I was and what was going on when I was first introduced to these terms; I remember a child playing with a ball that rolled away in the *migi* (right) direction, away from the park on the *hidari* (left).

Whereas we Europeans have a marked tendency to constitute ourselves, Cartesian-style, at the centre of our universes—left and right remain with me, but not with the landscape as I turn—the Aboriginal woman seems, to me, to see herself inscribed by/within the world, the constancy is with the landscape, which records her movement. Surprisingly, perhaps, the Aboriginal philosophy seems closer to our postmodern, the European is looking distinctly tired. In any event, locating—one of Bishop's fundamental activities—really doesn't appear to be the same kind of thing at all across these two cultures and, of course, we can produce similar stories for each of the other five.

Argumentation would seem, to me, to be fundamental to mathematics and not really covered by Bishop's list of activities. Alexander Luria (1976), conducting research in remoter parts of the Soviet Union in the 1930s, put syllogisms to his subjects and asked for conclusions:

> The following syllogism was presented. *White bears exist only where it is very cold and there is snow. Silk cocoons exist only where it is very hot. Are there places that have both white bears and cocoons?*
>
> Subject: Kul, age twenty-six, peasant, almost illiterate.
>
> 'There is a country where there are white bears and white snow. Can there be such a thing? Can white silk grow there?' …
>
> 'Where there is white snow, white bears live. Where it is hot , there are cocoons. Is this right?' …
>
> Where there is white snow there are white bears. Where it is hot there are white silkworms. Can there be such a thing on earth?
>
> (Luria, 1976; p. 105)

Luria presents what he understands as an analytic question—as do those of his subjects who have received some schooling and are literate—Kul, however, regards it as an empirical question and has no experiential basis on which to answer. Some time later, Basil Bernstein (1971) described a similar kind of difference that he explained in terms of 'speech codes'. Kul's approach might be regarded as an instance of what Bernstein referred to as *restricted code*, that is, an orientation to context dependent meanings. Bernstein reported a marked tendency of children from lower socioeconomic status backgrounds to be limited to this mode, grouping items of food, for example, in terms of the contexts in which they would be encountered in a meal. Children from professional, middle class families, on the other hand, would be more likely to group food items taxonomically—these are vegetables, these are cereals, and so forth. Bernstein attributed this to the possession of an *elaborated code*, which is to say, an orientation to universal meanings. What's more, the second group of children were generally able to switch to the restricted code when invited to group the food items in an alternative way.

Whereas the examples from Watson and Harris are pointing to differences between societies, Luria and Bernstein are identifying differences within a society albeit, in Luria's case, a society that he described as being in transition. All of these distinctions, however, are clearly related to fundamental differences in the sociolinguistic conditions of the respective societies or regions of society. James Gee and colleagues (2001) found resonant differences in the language used by teenagers from upper middle class and working class backgrounds in the US.

> The upper middle class teenagers are focused on knowledge claim, assessment, evaluation, their movement through achievement space, and the relationship between the present and the future. The working class teens are focused on social, physical, and dialogic interactions. [...] The upper middle class teenagers' interviews express, directly and indirectly, an alignment (and trust) among family, school, community, adult, and teen in terms of norms, values, and goals. The working class teens express, directly and indirectly, much less alignment (indeed, in many cases active disalignment) among family, school, community, adult, and teen in terns of norms, values, and goals.
>
> (Gee et al, 2001; pp. 183-4)

In particular, whilst the upper middle class teenagers tended to interpret their school activities in relation to future school and career possibilities and competition, their working class counterparts generally focused on the here and now, the solution of immediate problems, creating space for their own activity.

I'll mention just one more study revealing linguistic differences in terms of social class. This is Shirley Brice Heath's (1986) study of language use by lower socioeconomic status African American children and their families and by the mainly white teachers of their elementary school. Heath found that the teachers would, in their talk with their own children, often use exactly the same kinds of questions that they would used in the classroom, asking them about the colours that they were using in making a painting and so forth. Talk in the African American homes, however, did not include this kind of closed question. In contrast, originality in story-telling, the use of metaphor and so forth seemed to be far more highly valued. Needless to say, when children practised in creative talk responded in this way to the closed questions of the classroom, it was generally regarded as disruptive behaviour.

What I have been trying to demonstrate with these illustrations is that what we generally understand as mathematical and school knowledge is a long way from being universal and that some of its distinctiveness, at least, can be marked out in broadly sociolinguistic terms. We might say that If the school and its texts do not respond to this, then it will tend simply to reproduce the differences with which it is presented in its intake. I want to go on to explore some of the implications for the design of school mathematics texts and, in doing so, I want to begin to introduce some elements of an organisational language for thinking about this.

Creating Commentaries

Over the years, I have worked with people from many different backgrounds. What often happens in conversation is that people will tell me about their home countries. Here are a few examples; I shall use direct speech and inverted commas, although I am working from memory and am not claiming that these are the actual words that were spoken:

> ‘There are no regional or social class dialects in Australia; we all speak the same.’
>
> ‘Spanish is spoken in exactly the same way throughout Spain and the World.’
>
> ‘There are no social classes in new Zealand.’
>
> ‘There is no poverty in Greece, because extended families look after all of their members.’
>
> ‘Education under apartheid taught that there South Africa had no history before Jan van Riebeeck landed at Goede Hoop in 1652.’

I’ll come back to the last of these a little later. For now, I want simply to assert that the other four are simply incorrect. For the record: there are no societies of anything like the size of Australia or New Zealand (albeit that both countries have comparatively small populations) that do not exhibit socioeconomic stratification nor languages that do are not realised in different dialects; although poverty is constituted differently by different countries, no country escapes it; one estimate suggests that a third of the Greek population are near to or below the poverty line calculated on the basis of a monthly income per adult of €470.[1]

The point I’m making here is that whilst we are often regarded, by foreigners, as expert informants in respect of our own societies and whilst many of us might accept such attributions, most people simply are do not have such expertise, but are speaking from a particular location. I am not claiming that expert knowledge of a society is to be found in sociological or anthropological writing by sociologists and anthropologists who may have written about it. As Clifford Geertz (1988) has elegantly pointed out, ultimately, the claims of anthropologists are not to be validated by their having been ‘there’, in the field, but rather by virtue of them having been ‘here’, in the university and having learned to write anthropology. All commentaries are made from particular locations or perspectives; all are partial in terms of the information that they have

[1] See http://ipsnews.net/news.asp?idnews=40033.

available; all have involved the recontextualisation of that information in accordance with the interpretive schemes that they have deployed. I've illustrated this situation in Figure 1.

The double arrows in the diagram indicate that what is going on here is transactional. An interpretive scheme not only interprets texts and contexts, but also acts selectively on available texts and contexts and, in the sense that it can learn, is itself transformed in this transaction. Similarly, commentaries that are generated constitute new texts for interpretation and also inform the development of interpretive schemes.

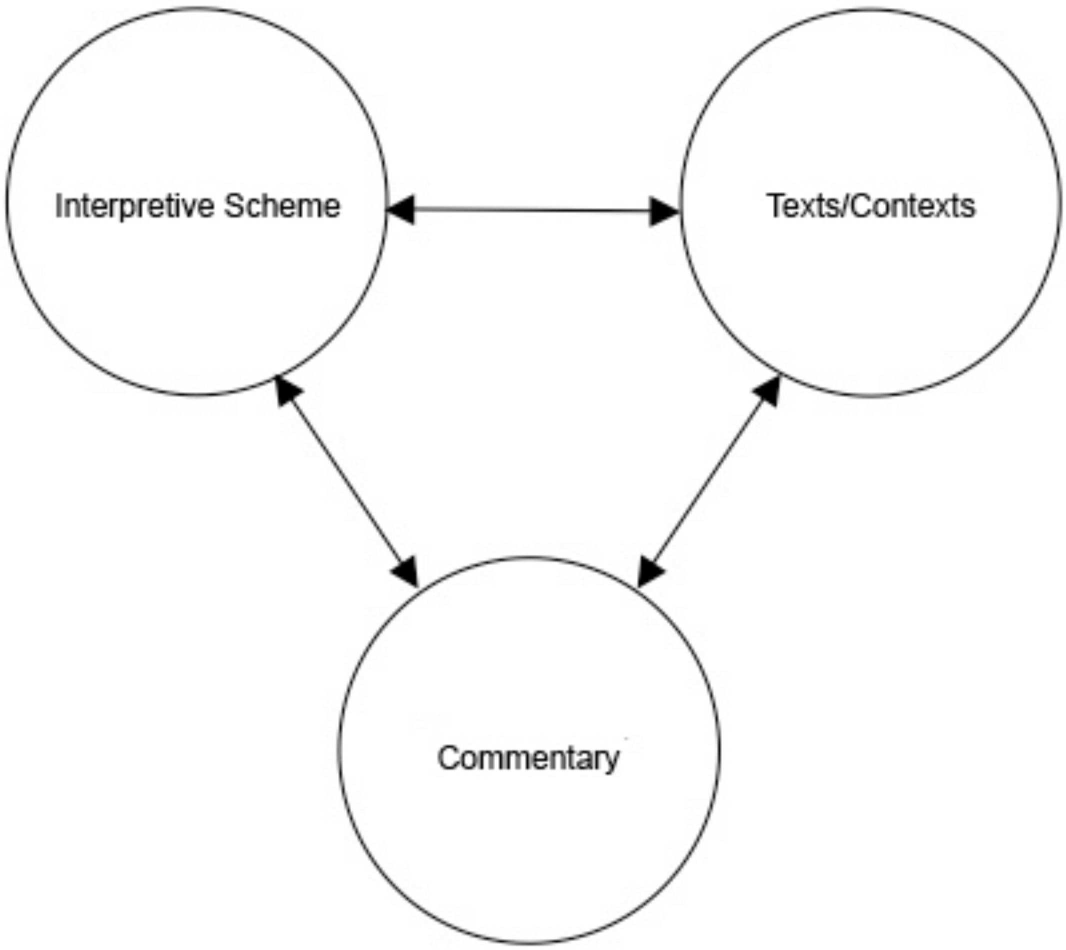


Figure 1
Schema of Interpretation

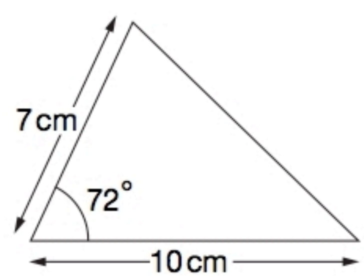
Mathematics and the 'Real World'

The schema in Figure 1 lays out the territory for consideration. Let me first look at the two primary school mathematics tasks in Figures 2 and 3. Now, the first task, Figure 2, is straightforwardly about mathematics and nothing else. The curriculum specifies that students must learn to draw simple geometrical figures accurately using ruler, compasses, protractor, etc and this is an example. The situation in Figure 3, however, is different. It's unlikely that there is anything in any mathematics curriculum that specifies birthday parties. What will be specified is that students should learn to perform basic arithmetical operations with whole numbers. The curriculum will probably also specify something to the effect that these operations should be encountered in various everyday settings etc, but it will probably not specify what these settings should be. If we look carefully at the second task, it seems clear that the activity is not really about birthday parties at all. It may be that a parent preparing for such an event will divide a cake into an appropriate number of slices, but it is very unlikely that they will engage in this kind of arithmetic in doing so. Danny may muse on the number of friends who will be coming to the party, of course, but he will probably have a list and will cross off those not coming and simply count the remainder. But whatever Danny and his parents do, they will not set out arithmetical tasks like this. As another example, just what domestic circumstances might necessitate this task:

> I want to make 12 cakes. If I know that 6 kg of flour is enough for 36 cakes, how much flour will I need?
>
> (http://www.primaryresources.co.uk/maths/pdfs/reallife3.pdf)

21 Here is a sketch of a triangle.

It is not drawn to scale.



Draw the full-size triangle **accurately** below.

Use a protractor (angle measurer) and a ruler.

One line has been drawn for you.

Figure 2
Task from 2006 Mathematics Key Stage 2 Test[2]

[2] Qualifications and Curriculum Authority, 2006

I want to suggest that school mathematics consists of, shall we say, non-arbitrary and arbitrary content. The non-arbitrary content—I call it the *esoteric domain*—is the collection of curricular items relating to number, shape and so on. It is then up to the textbook author and teacher to devise non-mathematical settings in which this non-arbitrary content can be realised. However, precisely because these settings are arbitrary, they can be transformed, recontextualised to conform to the particular, non-arbitrary mathematical priorities that are to be exercised, assessed and so forth. So, the interpretive schema of school mathematics may be said to cast its gaze on non-mathematical texts and contexts, producing commentaries that will serve as tasks in mathematics lessons. These commentaries constitute what I refer to as the *public domain* of school mathematics. So, the interpretive schema of school mathematics consists of, firstly, the esoteric domain of mathematics content and what I shall call *pedagogic theory* that enables the selection of suitable texts and contexts and their recontextualising as public domain content.

The Birthday Party.

Danny had been looking forward to his birthday. He had invited 34 friends around but 6 could not make it. He wondered how many would be left to join in the fun.

All the preparations had been made. Mom had made a cake, which had 24 slices in it. Dad ate half of the cake when Mom wasn't looking. Did that leave enough cake for Danny and his guests?

Mom decided to bake another cake with 36 slices. She could not work out how many slices there were altogether.

The day of the party arrived. It was decided that they would all sing 'Happy Birthday' and have some of the cake but only 8 people wanted any. They were all very greedy, how many slices did they get each.

Figure 3
Part of a Primary Maths Task[3]

I want to say at this point that the reverse of this situation is also possible. Preparing for birthday parties is a domestic activity and we can think of it as applying its own interpretive schemas. In deploying such schemas, a parent or child may consider what resources are at hand to help them accomplish the task and may well recall something from school mathematics. However, now it will be

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[3] From http://www.primaryresources.co.uk/maths/maths.htm.

the mathematics that is arbitrary, leaving it open to recruitment or not as is convenient.

Not only are school mathematics and domestic activity distinct regions of practice, they are quite fundamentally different modes of practice. Bernstein (2000) distinguishes between what he calls vertical and horizontal discourses. A Vertical discourse is that which is acquired in formal schooling and involves progression over an extended period of time resulting in the acquisition of context independent knowledge—reminiscent of the elaborated code, which was presumably its origin in Bernstein's thinking. A horizontal discourse, on the other hand, is acquired in the context of its deployment. Most domestic practices are like this second kind of discourse.

Bernstein's distinction is a helpful one and I want to introduce a related concept, *discursive saturation*. Mathematics is an example of a practice that is heavily dominated by explicit principles; I refer to this as high discursive saturation (DS^+). Mathematics pedagogy generally entails encouraging students to make explicit the rationales for their actions and the bases of their understanding. This is from the introduction to a primary mathematics scheme.

> A teacher interested only in answers might have seen this, from seven-year old Mary, and just marked it right:
>
> 25 - 18 = 7
>
> Fortunately, Mary's teacher was interested in processes, encouraging the children to write everything down. This is what she actually got from Mary.
>
> 25 - 18 is 5 take away 8 is 5 steps forward and 8 steps back, that's -3. 20 take away 10 is 10. 10 and -3 is 7. 7 is the answer.
>
> From this she learnt that Mary had a good 'feel' for numbers and an understanding of place value and negative numbers (although we would not necessarily expect the latter at this level, nor does the National Curriculum require it).
>
> (*Nelson Mathematics*[4]; p. 18)

I am not entirely clear as to just what a "good 'feel' for numbers" means, here. But the practice of requiring students to be explicit is, of course, a strategy that is consistent with an explicitly principled practice. Domestic practices, by contrast, very substantially involve tacit knowledge. It's not that it is impossible to produce a discourse on cooking that draws on knowledge of organic chemistry and so forth, but rather that this is generally not the way in which cooking skills are practiced and acquired. These are low discursive saturation (DS^-) practices.

The DS^+ nature of much—not all—of mathematics entails that it is very substantially an analytic practice, yet there is an extent to which this is undermined by the incessant reference by school mathematics to the empirical world. I have already noted this incompatibility in respect of Luria's syllogism. Syllogistic argument is, of course, the standard form of mathematical reasoning.

[4] The scheme is published by Thomas Nelson & Sons, Walton on Thames and is dated 1992.

However, syllogism immediately breaks down when it is projected into the empirical. Thus:

> All those born within the sound of Bow Bells are cockneys;
> Dowling was born within the sound of Bow Bells;
> Therefore Dowling is a cockney.

This is fine if we leave it in the analytic. But as soon as we try to make sense of it in the empirical, we are bound to seek clarification. How is ‘within the sound’ to be defined? Does it depend on the weather, ambient noise, the hearing of the listener? Do the bells actually have to be ringing at the time of the birth and be heard, by whom? Is it relevant that the bells were not sounding at all at the time of Dowling’s birth as they had been silent since the start of World War II until 1961?

The *Nelson Mathematics* scheme seems to propose that the origins of mathematics are empirical, thus:

> - children need concrete experiences if they are to acquire sound mathematical concepts;
> - like adults, children learn best when they invent and make discoveries for themselves;
>
> (*Nelson Mathematics*; p. 5)

Such empiricism seems to dominate much of school mathematics and it’s worth mentioning that the text seems to imply that children will not have concrete experiences unless they are provided for at school. Here is an account of a lesson:

> Some time ago a beginning teacher described to me an incident in a mathematics lesson that she had been teaching. The year seven students had been constructing triangles to measurements that had been put on the board: 8 cm., 7 cm., 6 cm. and so forth. As a final example, the teacher had included 6 cm., 3 cm., 3 cm. Having arrived at this example, one of the students announced that this was not a triangle. Another student - a girl - responded:
>
> ‘Oh yes it is, and what’s more I’ve drawn it.’
>
> The teacher invited her to draw the triangle on the board using a board ruler. The girl first constructed a line of length 6 units horizontally. She then tilted the ruler very slightly and drew a 3 unit line from one end of the first line and finally joined this to the other end of the first line by constructing another line of similar length.
>
> ‘But those lines are more than 3 cm’, observed one of the boys in the class. A number of the others agreed with him.
>
> ‘OK, I’ll make them smaller’, the girl said and duly constructed a somewhat flatter triangle.
>
> But the other students remained unconvinced. One of them tried a different approach,
>
> ‘You can’t draw that triangle, because 3 cm and 3 cm make 6 cm so the two short sides have to be on the same line as the long side or they won’t join up’.
>
> A debate ensued with most of the class joining in. Eventually, everyone had sided with the sceptics. Everyone, that is, except the girl who had drawn the ‘*impossible*’ object on the board. She remained, throughout, stubbornly attached to her claim that she had in fact achieved the impossible and that her triangle was to be allowed.
>
> (Dowling, 2001; p. X)

This, I think, was a first year secondary classroom, but it could well have happened in a primary school. Given my comments on the analytic nature of mathematics, it will come as no surprise that I side with the girl who had drawn the impossible triangle. Referring back to Figure 2, the marker of this task will presumably make use of a template triangle inscribed on a transparency. This can then be overlaid on the student's drawing for assessment purposes. But the marker will have to allow a margin of error, because measurement is not and can never be perfect. Now if the teacher in the classroom activity used the same method, it would be perfectly possible for the girl to draw a triangle that approximated, but that was not, a straight-line segment. By turning mathematics into an empirical practice, the teacher's pedagogic theory had recontextualised mathematics itself.

I do not intend to claim that, in its origins, mathematics has no connection with the empirical; that would be absurd. But, despite all of its referral to the 'real world'—which I take to mean the world beyond mathematics *per se*—the mathematics curriculum, the non-arbitrary esoteric domain, is primarily constituted as self-referential, self-contained. The 'real world', wherever it appears in a mathematics lesson or test must be made to conform with abstract mathematical structures. Furthermore, the fact that it has taken an awfully long time for mathematics to abstract itself from the empirical in the way that it has—numbers were adjectives for thousands of years before they ever became nouns, for example—ought to suggest that the move from public to esoteric domain is not an easy one, though it might be easier for some than for others. If we can generalise a little from the examples at the beginning of this paper, we might expect those children having backgrounds in professional middle class, European homes to do rather better than others at privileging the abstract even when mathematics is presented encoded in the concrete. This is not because working class children or children from non-industrialised societies are defective, but because contemporary European schooling privileges self-referential, DS^+ practices. This was the message of the 'new sociology of education' in the 1970s; oddly, perhaps, one of its principal exponents, Michael Young, (Young, 1971) now adopts a rather contrary view (Young, 2000, 2004); but then the dynamic imperative of the university that has us perpetually struggling to say new things tends to ensure that we quickly leave behind what was new yesterday; of course, some of the new things have been borrowed from the day before yesterday; in some respects, academic work is like fashion.

It might be (and has been) argued that the privileging of European school mathematics in European schools, though it may be more consistent with professional middle class discourses than with working class discourses, is nevertheless justified because these are the dominant discourses of contemporary European society. Bernstein effectively said as much in the mid-seventies:

> It is an accepted educational principle that we should work with what the child can offer: why don't we practice it? The introduction of the child to the universalistic meanings of public forms of thought is not compensatory education—it is education.
> (Bernstein, 1974, p. 199)

I am not necessarily challenging this, but I do want to look a little more closely at the basis for such a claim in respect of the mathematics curriculum, that is, that the universalistic (abstract and self-referential) meanings of school mathematics are in some sense necessary in order to function in contemporary European society. Here is Bridgid Sewell in a report commissioned on behalf of the Cockcroft Report into mathematics teaching (Cockcroft, 1982):

> Percentages play an ever increasing part in the dissemination of information, both through the news media and from central government. An understanding of the national economy assumes a sophisticated comprehension of percentages, as does much of the discussion about pay rises. For the shopper, the ability to estimate 10 per cent can be a valuable 'key' to checking other percentages—even if a precise answer seems too difficult. Since the currency became decimalised, it is a trivial matter to work out 10 per cent of a sum of money, and this can easily be used to estimate other percentages. Those who lack the skill even to calculate 10 per cent are surely handicapped when attempting to understand the affairs of society.
> (Sewell, 1981; p. 17)

There may have been a point in time when this was the case (though I doubt it). Now, however, it is generally no more necessary for us to be able to calculate the effect of a ten percent reduction on a price than it is for us to be able to calculate our mortgage repayments. Our culture incorporates calculators in the forms of price labels, tables, shop assistants and so forth. It is necessary for *some* people to be able to deploy the appropriate technology in order to perform such calculations, but not for everybody to be able to do so. I have to report that I virtually never perform any kind of calculation in any context. When it comes to shopping, this may be because a full professor's salary, though very far from magnificent, at least allows me not to have to worry that I won't be able to cover the cost of the goods in my supermarket trolley or the increase in my mortgage repayments when the rates go up. But there is also ample evidence that shoppers who do need to shop economically are able to do so without any assistance from school mathematics, devising context specific tactics in making best-buy decisions (Lave, 1988; Lave et al, 1984). I have noticed, also, that many of the price labels in major supermarkets in the UK have, for some time, included the price per unit of measurement.

Sewell's proposition that 'An understanding of the national economy assumes a sophisticated comprehension of percentages, as does much of the discussion about pay rises' is meaningful only if either a sophisticated comprehension of percentages were a sufficient condition for understanding the economy or if complementary skills are to be provided elsewhere. In fact, neither is the case as is illustrated by one of the questions that Sewell herself put to her interviewees:

> On the news recently it was said that the annual rate of inflation had fallen from 17.4% to 17.2%. What effect do you think this will have on prices? (If answer 'none') What do you think ought to happen if it had fallen to, say, 12%?
>
> (Sewell, 1981; p. 33)

Clearly, Sewell's own understanding of the economy is somewhat limited, despite her (presumably) sophisticated comprehension of percentages. The line of causality in this question is reversed, changes in prices having an effect on the rate of inflation, rather than the other way around and, what's more, the inflation index is always retrospective. But it's also questionable what inflation rates tell us about what has been happening to prices; here is a piece from the 'Money' section of *The Guardian*, a UK daily newspaper quite widely read by teachers and other middle class professionals.

> Government data puts annual food inflation at 6.6%. But in the malls of Britain shoppers would be quick to say the number-crunchers are having a laugh. At Asda, a dozen free range eggs cost £1.75 in May last year. Now the price tag is £2.58—a rise of 47%. In Sainsbury's, 500g of pasta has gone up from 37p to 67p—an 81% increase. Bread is up by 20%, English cheddar by 26% … the list goes on (see page 3).
>
> (*The Guardian*, 17th May 2008)

What is important is not so much an understanding of the economy—who has that?—but an ability to maintain a household budget and, as I have indicated above, that is done effectively (or not) without the assistance of school mathematics. If we are engaging in political discourse, then we may take a view on inflation. Is it bad, because prices go up and wages generally don't seem to go up as much. Is it good, because, although wages don't keep up with inflation, they nevertheless go up, generally entailing that the proportion of our income that we spend on mortgage repayments goes down, leaving us with a net effective increase in spendable income; we're not often told that, are we, but I didn't calculate it; I experienced it. Having had an extended period of low levels of inflation, I've found that I'm not getting any better off as I used to in the 'bad old days'.

The Guardian also has tips for coping with price inflation:

> Don't casually buy ready made things that take moments to make at home at a fraction of the price, […] "Never buy pre-made pasta sauces (£1.50 a jar) as these can [be] whipped up in the same time it takes to cook the pasta—at a fraction of the cost. Fry an onion, add some tinned tomatoes, a few herbs, a dash of wine—and in eight minutes you're done."
>
> [Allegra] McEvedy, who co-owns the Leon chain of restaurants, says it will not only taste better, it will have less salt and sugar often used to bulk out factory-made sauces.
>
> *The Guardian*, 17th May 2008)

I would certainly agree with Allegra that the sauce is likely to taste better, though I would also suggest including a little shoyu (increasing the salt content), cayenne or black pepper and, if one wants to sweeten the sauce by marmalising the onion, then it's going to take a great deal longer than eight minutes. On the price, checking with some recent till-slips, a tin of tomatoes costs 42p (what if I'd bought organic?) and a large onion 30p (but you have to buy three). I believe (from memory) that a small pack of fresh basil costs about 70p. Now assuming that I have the wine (open) and am happy with just one herb, then I've made a saving of 8p; the particular fraction of the cost in this case (not counting the cost of the wine, additional cost of cooking over heating, or my time) being 95%, but it's only this calculation that has required any knowledge of percentages. Unless one is cooking and buying in bulk (like a restauranteur), in which case many of us would have storage difficulties, then I have no doubt that cooking from the basics at home generally costs more than buying prepared food and one frequently does have problems buying the right quantities (hence recent reports of large amounts of food being wasted in the UK[5]). I didn't need to do any calculations to know this; during periods when I have relied on buying prepared food, my shopping bills have been distinctly lower than they are when I do my own cooking. One doesn't do it because it's cheaper, but because it tastes better and gives one more choice.

And, as an aside, because my partner of the last eight years is Japanese and lives and works in Japan, we are together for a little under half of each year. So I spend a good deal of the time living on my own. Comparing this with my experience of my previous marriages, I find that, despite the fact that I always did my share of housework etc, I am spending far more time on housework and other domestic responsibilities (dealing with banks, stores etc) than I ever used to and also that buying food for one is far less economical than buying for two. But this, as with home cooking, is a lifestyle choice and not a financial one. Where household income is such that financial criteria do come to the foreground, most people manage at least to develop tactics for quantification. It's poverty that drives them to the pawn shops and loan sharks (including the increasing number now advertising on TV) and not a lack of mathematical skill. For the overwhelming majority of people, mathematics is not of any use to them.

Domains of Practice

I want to look at a type of task that is very common in primary mathematics, the 'word problem'; an example is given in Figure 4. Now this differs from the public domain task in Figure 3 in an important, though not dramatic way. Essentially, the Birthday Party task, by providing some extended background to the setting and implying possible (if rather stretched) motives for the calculations, is suggesting that this is a 'real world' problem, that it is the kind of thing that Danny might do spontaneously in relation to his birthday party, perhaps. Furthermore, the language of 'The Birthday Party' is more or less consistent with the language that might be used in relation to the birthday party setting. The Figure 4 task, however, provides minimum information about the setting, simply encoding a mathematical structure in familiar language. Mathematically, the task might be expressed as follows:

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[5] http://www.guardian.co.uk/environment/2008/may/08/food.waste.

$$n \propto t$$

$$t = 1 \Rightarrow n = 17$$

$$t = 8 \Rightarrow n = ?$$

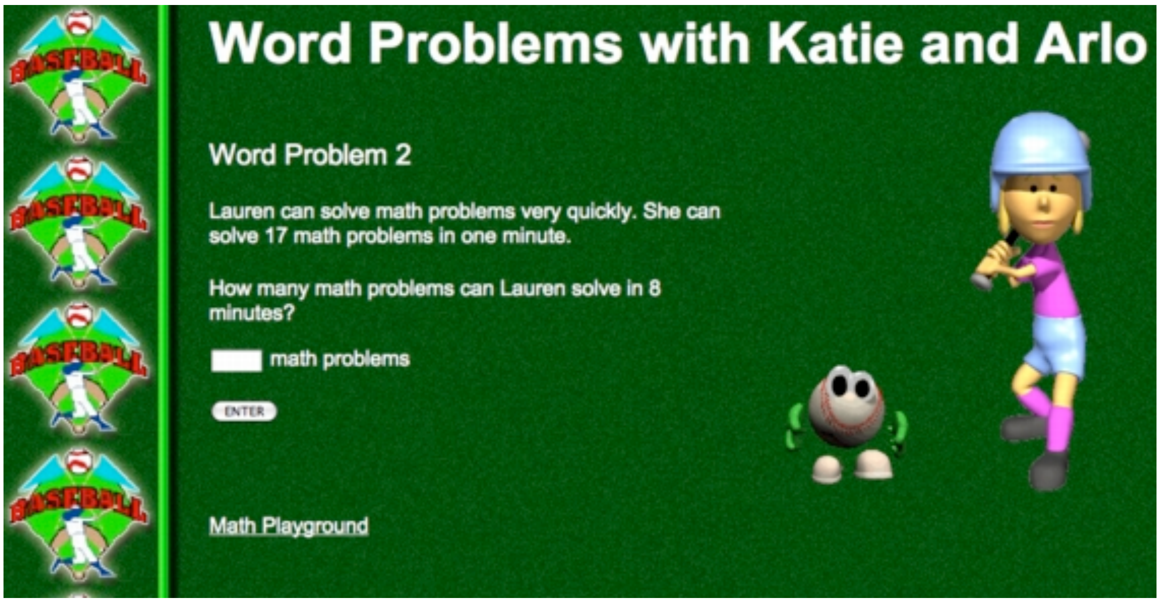

Figure 4
A Word Problem[6]

Of course, the first line of the mathematical formulation, or some other relation between the number of problems and the time taken to solve them, has to be presumed, otherwise the problem is insoluble. In terms of the nominal setting—Lauren solving math problems—this presumption is, of course, absurd; the time taken to solve a math problem depends upon the problem and might be expected to vary even when the same problem is repeated because of other factors, such as fatigue (or, alternatively, the benefits of getting 'into' the activity). But this doesn't matter. The setting is as irrelevant as the movie images of Katie and Arlo (which is which?) playing baseball (if you input the correct answer to the problem into the website, the hitter starts to run) or the image of the wrapped present in 'The Birthday Party'. Although the student will not be expected to know the mathematical language that enables the decoding of the mathematical formulation of the task (*n* is directly proportional to *t*, if *t* is 1 then *n* is 17, if *t* is 8, what is the value of *n*), they will also be unaware of any alternative mathematical relations between the two variables, so the lack of any explicit coding of direct proportionality in the task doesn't affect their ability to solve the problem. The

6 http://www.actionmath.com/Katie2/Katie2wp2.html

teacher may encourage understanding of proportionality using the language of the setting, thus:

> Lauren solves 17 problems in each minute.
> How many problems has she solved after two minutes?
> How many after three minutes?
> How many after five minutes?
> How many after 20 minutes?
>
> Suppose we give her some more difficult problems. She can do only 12 of these in each minute. How many in ten minutes?

What the teacher would be doing is mapping out direct proportionality in everyday language. Now thinking about the situation in this way enables me to extend my esoteric/public domain schema; I'll do this analytically. I want, first, to distinguish between the *expression* in a message (it may be only one word or more, or it may be an image, etc) and what the message is taken to refer to, its *content*. I want now to propose that each of these may be more or less strongly associated with mathematics *per se*. Thus, the expression in the mathematical formulation of the task in Figure 4 is strongly associated with mathematics; I'm going to say that it is strongly *institutionalised* (I^+). The content of this formulation is also I^+, the referents are mathematical objects, numerical variables and relations. By contrast, the expression in 'The Birthday Party' is not really associated with mathematics at all; it is weakly institutionalised (I^-) and its content is also I^-, referring to slices of cake and so forth. As I have already indicates, I refer to these two kinds of text as esoteric and public domain respectively.

The text in Figure 4, however, is different. Whilst its expression is like 'The Birthday Party', that is, I^-, its content is the same as that of the mathematical formulation, I^+. I shall refer to this kind of hybrid text as *expressive* domain. These three domains are shown in Figure 5, which also reveals a fourth domain, the *descriptive*; I'll return to this later. As well as word problems, expressive domain texts include pedagogic metaphors. A very familiar example in the primary school curriculum appears in Figure 6. This is a Grade 4 mathematics test item from the Trends in International mathematics and Science Studies. I gloss this metaphor, 'a fraction is a piece of cake'. Now one problem with this metaphor is illustrated in Figure 7. The answer to the question in Figure 6 is, of course, '1 and 2', because both of these represent the fraction $^3/_4$ (the same as $^6/_8$). However, if the first diagram in Figure 6 represents $^3/_4$ because three slices out of four are shaded, then a perfectly reasonable solution to the sum in Figure 7 is $^6/_8$ because the two cakes have eight slices, six of which are shaded—a common error.

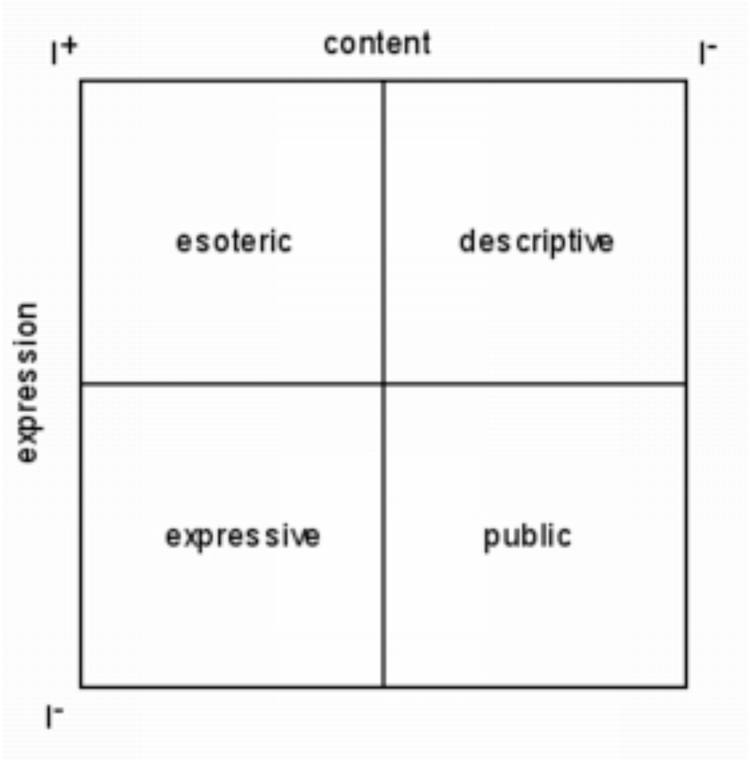

Figure 5
Textual Domains

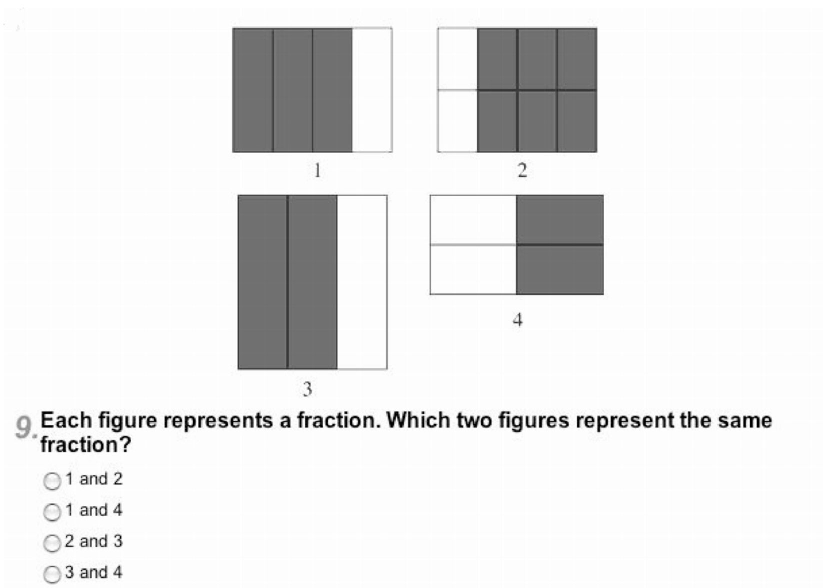

Figure 6
TIMSS Grade 4 Mathematics Test Item[7]

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7 See http://nces.ed.gov/timss/.

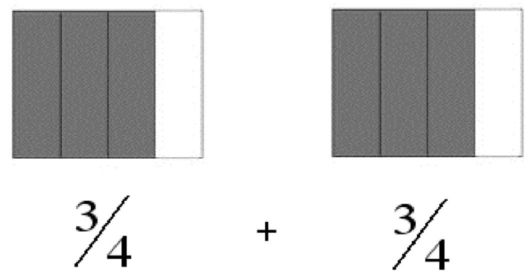


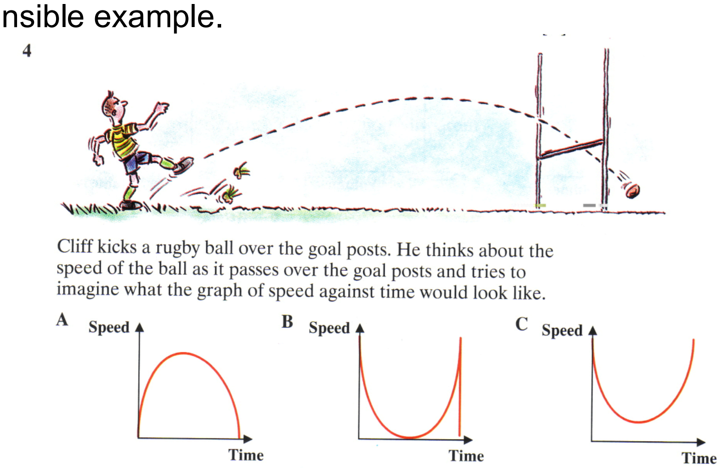
Figure 7
Adding Fractions or Pieces of Cake?

This solution is, of course, incorrect, but it is not immediately apparent to someone operating exclusively in the expressive domain why this is the case. In order to be able to grasp the mathematics that has generated the expressive domain text, there has to be a move into the esoteric. Mathematically, a fraction is not a piece of cake, it's a number and, in the case of $^3/_4$, it's a number between 0 and 1. The denominator names, however, have a place in the language of the everyday, so we do talk about dividing a cake into quarters or eighths and so forth. We might imagine that people learn about this kind of activity without the help of school mathematics and that's possibly why the expressive domain has such an appeal as a reservoir of metaphors for mathematics. The dangers, however, are clear, if the student of mathematics is left in the public or expressive domains and not provided access to the esoteric. One of my doctoral students, Jeremy Burke, has described those domains that are outside of the esoteric as oubliettes!

The fourth domain in Figure 5 is the descriptive. Here, the mode of expression is I^+, but the content is I^-; this is the opposite configuration to that of the expressive domain. In the descriptive domain, the language is mathematical, but it is being referred to non-mathematical content; this is the domain of mathematical modelling. Now, clearly, there are areas of activity that recruit mathematical structures to 'model' non-mathematical texts and contexts. As a graduate of physics I am very familiar with the mathematical modelling of the universe and as a sociologist I am very familiar with the statistical modelling of society. I suspect, however, that, numerically, there are actually very few people involved in this kind of activity and those who are will receive extensive training. Most of us, I want to contend, do not engage in this kind of activity at all (though I've always wondered whether mathematics teachers have a tendency to take their mathematical modelling outside of the classroom).

Mathematical modelling has recently been given a very high profile on the English National Curriculum. Even though it is from the secondary rather than the primary curriculum, I cannot resist reproducing an example from the textbook produced on behalf of Edexcel, the company that is dominant in producing and managing public examinations in England. The example, which was pointed out to me by the doctoral student mentioned above, is shown in Figure 8.

As with some of the other examples introduced here, the mathematising of this setting is absurd. Cliff's attempts to imagine the shape of the graph of speed against time of the rugby ball alibi, what is quite clearly a ridiculous activity; certainly, if he was thinking about graphs he would certainly have missed the shot at goal! The best answer is the third graph (assuming that the horizontal axis marks zero speed, though there are no scales) the second graph seems to suggest that the speed of the ball is zero at its highest point—it isn't, though the vertical component of the velocity is—and that the ball undergoes an infinite deceleration, presumably when it hits the ground and stops dead. The point is that, whilst mathematical modelling can be fun—if you find mathematics fun—it is almost always a redundant activity from the point of view of the context that is being modelled and that might explain why Heinemann/Edexcel are unable to find a more sensible example.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Cliff kicks a rugby ball over the goal posts. He thinks about the speed of the ball as it passes over the goal posts and tries to imagine what the graph of speed against time would look like.

(a) Write down the letter of the graph which best illustrates the movement of the ball.

(b) Give reasons for your answer to part **(a)**.

Figure 8
From Heinemann/Edexcel Higher Tier Mathematics

These examples are intended, amongst other things, to illustrate that, whilst mathematics curricula may well include references to mathematics and the real world and modelling, that which drives school mathematics is the esoteric domain of mathematical content that is, to all intents and purposes, self-referential and closed. Whilst school mathematics might recruit a whole range of non-mathematical texts and contexts for pedagogic (and, indeed, ideological) purposes, this 'use' of mathematics is by and large confined to the school; for the vast majority of people—including many of those of us who are university professors—mathematics simply is not useful and, when it is, it is recontextualised mathematics, that will generally tear it out of its school setting and make it other than it is.

The illustrations from the beginning of this paper are intended to demonstrate that the self-referential esoteric domain of European school mathematics is not the natural destination of ratiocination, but a peculiarly European product that resonates more in the practices of some sociocultural groups than in those of others and students from these other groups are unlikely to find their own way to this domain, however obvious the route may appear to their teachers. To expect them to do so would be rather like expecting them to learn Japanese by watching silent Japanese movies!

What students may tend to learn is to combine esoteric and public domain text inappropriately; to presume incorrect models. I once started a mathematics lesson by saying,

‘Suppose I have two glasses of water, both at 20 degrees and pour them both into the same bowl; what will be the temperature of the water in the bowl?’

’40 degrees.’

‘Good! Now supposing I pour another three glasses of water—all at 20 degrees—into the same bowl; what temperature now?’

‘100 degrees.’

‘Wonderful! And what happens at 100 degrees?’

‘Water boils.’

‘Excellent! So now we know how to heat up water without a kettle. But there’s more. I think 20 degrees is too warm for water if I’m going to drink it. So ‘m going to take one glass and pour the water into two glasses; what temperature is the water in each glass?’

’10 degrees.’

‘Still a little warm, I think, so what shall I do?’

‘Pour one of the glasses of water into two more glasses.’

‘And the temperature will be?’

‘Five degrees.’

‘Just right. Now, when you go home this evening, you can explain to your parents how to cut down on their electricity bill.’

…

‘This isn’t right, is it, sir?’

Indeed it wasn’t.

Mathematical Discourse and Skills

I have been suggesting that mathematics is a DS^+ practice, that it is a practice that tends to make its principles explicit. But this is only partly the case. Figure 9 Shows a screenshot from an online game published on a school website.[8] The two ghosts appear from the haunted house that you can see behind them. They each have a number. In this particular setting of the game, you blast any pair of ghosts whose numbers sum to fifty and get a point; you lose a point if you blast innocent ghosts whose numbers don’t sum to fifty. The ghosts appear only for a

[8] The game is at http://www.oswego.org/ocsd-web/games/ghostblasters2/gb2nores.html, which is on the Woodlands Junior School site at http://www.woodlands-junior.kent.sch.uk/index.htm.

very short time, so you have to be quick. The game is played by two players (who can type in their names); Player one blasts using the Z key and Player two uses the M key.

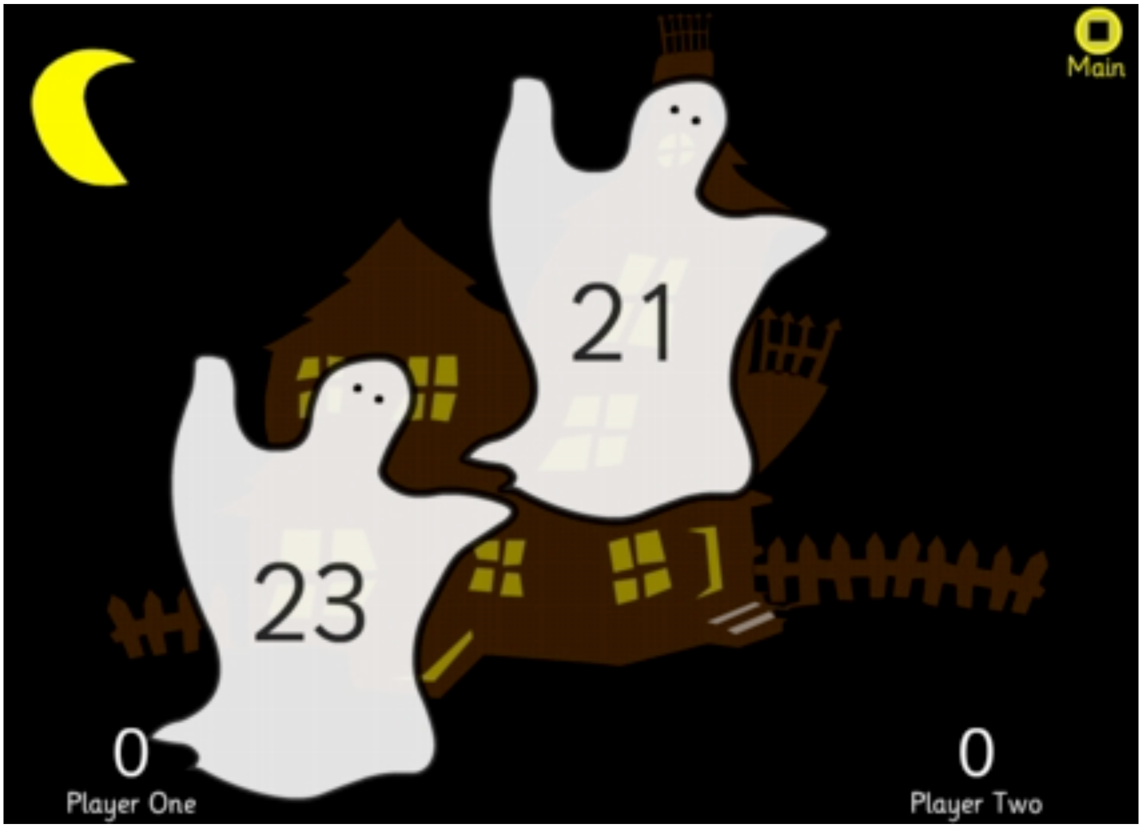

Figure 9
'Ghost Blasters II'

This game is not about the acquisition of a principled, DS^+ discourse. Rather, it is about the development of a skill that, in this case, is both sensorimotor (you have to move fast) and cognitive (you have to think or recall fast). This is DS^-. Of course, a good deal of the school mathematics curriculum involves this kind of activity. With tasks relating to DS^+ activity, such as Mary's subtraction of 18 from 25 and subsequent explanation, or the triangle drawing activity that was supposed to lead to the realisation that 6 cm, 3 cm and 3 cm are not the lengths of the sides of a possible triangle, the discussions are concerned to develop competence in the student; the teacher cannot be interested solely in the answer—the students performance. Performance, however, is precisely what is important in 'Ghost Blasters'. Now, of course, different students will have different tactics for achieving their performance. Some might perform calculations very quickly, using one or more of a range of possible approaches; others might, after a while, manage to memorise all of the whole number pairs that sum to 50. Some of the approaches that the students use may be idiosyncratic, which is to say, not institutionalised—explicitly taught—by the curriculum. Mary's approach in subtracting 18 from 25 might be thought of as idiosyncratic. The teacher may of course be interested in these idiosyncratic approaches, but the point of the 'Ghost Blasters' task is to develop and hone a skill.

Now, in this brief discussion of 'Ghost Blasters' and Mary's calculation, I have introduced the basis for another schema from my organisational language. My first dimension is discursive saturation, which may be DS^+ or DS^-. The second is the level of institutionalisation, which can also be high (I^+) or low (I^-). The product of these two variables gives rise to the shaded cells of the schema in Figure 10. My contention is that this schema provides a helpful way of thinking about the mathematics curriculum as a whole and suggests the range of strategies and resources that might be deployed in respect of different aspects of the curriculum.

	Institutionalisation		Pedagogic Strategies	Pedagogic Resources
	Formal (I^+)	Informal (I^-)		
DS^+	*discourse*	*idiolect*	*specialising* *generalising*	*principles*
DS^-	*skill*	*trick*	*localising* *articulating*	exemplars
	Pedagogic	Exchange		
	(Re)producing Activity			

Figure 10
Practical Strategic Space

Discourse constitutes the way in which I initially described mathematics, that is, as an explicitly principled, self-referential practice. In developing this mode of practice, we must arrive at the principles and, as I have argued, as these are culturally arbitrary, the students will not arrive at them on their own; sooner or later they must be formulated for them. I have referred to two pedagogic strategies, *specialising* and *generalising*. These are opposing strategies, specialising narrows the focus by exploring principles as they relate to a particular class of object. We might think, for example, of the properties of whole numbers or of triangles rather than of numbers or plane figures more generally (generalising).

As I have indicated, *skills* are DS^- practices that are not susceptible to principled elaboration. The pedagogic resources available are *exemplars* rather than principles, the teacher must show or point at what they mean, although, of course, they may describe it in various ways: principles may reveal why an arithmetic algorithm works, but it will not enable a student to acquire dexterity in deploying it. Because skills are not principled, their territory is delimited not by specialising, but by *localising*: we will practice the recall of number bonds.

Similarly, the territory is expanded not by generalising, but by *articulating*: we will practice the algorithms for the four rules of number.

Both discourse and skill are formal requirements of a curriculum, they are strongly institutionalised by and in it. However the teacher's approach might be presented as student-centred and predicated on constructivist epistemology, discourse and skill are non-negotiable and the teacher, as the surrogate author of the curriculum, retains the principles of evaluation of students' performances. I refer to this kind of activity as *pedagogic*. In pedagogic practice, *authority* is retained by its author.

However, teachers might also be interested in encouraging students to develop their own approaches to mathematical situations and problems. Sometimes this is planned to be *en route* to discourse or skill. But sometimes, given differences between students and between contexts, idiosyncratic methods are more effective. If idiosyncratic, or informal methods are genuinely encouraged and not 'corrected', then we have a different kind of classroom practice. Now, authority is being handed over to the student, the audience of the curriculum. I refer to this kind of activity as *exchange*. In exchange activity, authority is transferred to the audience.

Figure 10 incorporates two modes of informal method: the *trick* is DS^- and corresponds to the formal skill; *idiolect* is DS^+ and corresponds to discourse. There is a sense in which both modes were to be encouraged in the original conception of the mathematical investigation, in the sense that students would be encouraged to produce mathematics that was not on the curriculum and that might well be new to the teacher as well as to the student. Mary's approach to subtraction might, in the way that she has formulated it, be regarded as idiolect—it certainly seems to related to principled knowledge—but it is easily reconciled with more conventional formulations. The informal column of Figure 10 puts the hard question to the mathematics teacher: at degree zero, does the student ever really have authority in the classroom; is it not always the case that, ultimately, whatever they produce must, if they are to be accredited, be reconciled with the curriculum, must be recontextualised.

Authority and Interaction

Whereas much of what I have been saying can relate easily to textual material, the final point in the previous section has made it clear that it is the text in action that is fundamental to the understanding of what is going on or what might go on in mathematics classrooms. I want to introduce, very briefly, two more schemas that may open up some possibilities for interrogating the classroom.

The first concerns authority. I have made a distinction between pedagogic and exchange activity. In the former case, authority is vested in the author of the curriculum—the teacher, in the classroom. But what form does this authority take? I want to propose that authority may be located either in the person of the author or in the institutionalisation of the curriculum, or in both or, of course, in neither, in which case we have an exchange form. The schema that results from this conception of the situation is presented in Figure 11. Within this schema, the *traditional* mode of authority brings together the singular curriculum author and the strongly institutionalised practice. This, I suppose, is the traditional image of the teacher, who has received training in a particular field and it is this training—not easily transferred to anybody else—that qualifies them to teach. By contrast, the production of official curricula or standard textbooks and tests seems to take the authority away from the teacher and locate it in a practice that is codified in official documents. To the extent that the curriculum is presented in a way that is accessible beyond the teaching profession—and this seems increasingly to be the case, in one way or another—then the teacher is constituted as simply the agent of the curriculum and is replaceable by any other agent. This is bureaucratic authority. Alternatively, the teacher may seek to reclaim their authority by challenging (or ignoring) the *bureaucratic* form. Such a teacher may feel, for example, that keeping rigidly to the curriculum will prove counterproductive in respect of students' performances and, insofar as their own performance is judged solely by how the students do on the tests, they may choose to teach in an idiosyncratic way and assert or demonstrate that it is effective. This is *charismatic* authority.

	Field of Practice	
Category of Author	Open	Closed
Closed	*charismatic*	*traditional*
Open	*liberal*	*bureaucratic*

Figure 11
Authority Strategies[9]

To the extent that the teacher genuinely yields authority to the students in exchange mode, we have the *liberal* pedagogy proposed by Piaget (1995) and by a good deal of constructivist discourse. Though not, interestingly, by Vygotsky (1978, 1986), whose 'scientific' concepts derive from didactic instruction. It is clear that the pedagogy that might be associated with 'situated cognition' was also, in its formulation by Lave and Wenger (1991), was also didactic and so reproductive of cultural practices. In his subsequent move to management consultancy, Etienne Wenger (1998) helpfully introduced negotiation into the frame, though in rather different contexts from the strongly institutionalised forms of the earlier work.

It will be apparent from my comments in the previous section that I remain somewhat sceptical about the existence—or even the possibility—of the liberal mode in the context of the primary school, though it can certainly be found at the other end of the educational system. In the supervision of doctoral theses in

[9] I have the three terms, charismatic, traditional and bureaucratic as applied to modes of authority from Max Weber (1964), but have used them in a resonant, but fundamentally different way.

educational studies, for example, originality is an explicit requirement and is sometimes achieved in rather more than a mechanical way. Scepticism notwithstanding, the schema in Figure 11 asks the teacher to reflect on just where authority is being vested in any given situation: a charismatic move can, after all, corrode the most liberal of intentions.

Finally, in this section, I want to reflect on interactions. Any interaction may be considered to occur between similars or disimilars. In the context of the classroom, the most obvious way of thinking about this would be to construe peer interaction between students as constituting interaction between similars and interaction between teacher and students as interaction between disimilars. Naturally, though, students do differ amongst themselves (as I have been at some pains to point out earlier in this paper) and, after all, Vygotsky's (1978, 1986) concept of the zone of proximal development allows for interaction between peers of differing levels of achievement as well as between student and teacher.

The second dimension of my final schema concerns the way in which an interaction is set up in terms of whether it is designed—by some mode of authority—to aim at closure, which is to say, the achievement of coherence amongst the participants, or openness, in which case there is no authoritative pressure on participants to accommodate to another position. These two variables give rise to the schema in Figure 12.

	Target	
Interaction Between	Closure	Openness
Similars	*equilibration*	*exchange of narratives*
Disimilars	*hegemony*	*pastiche*

Figure 12
Interactive Modes

Now the conventional image of the classroom would seem quite definitely to be *hegemony*. The teacher knows, the students do not and the target is the closure on the teacher's knowledge, or at least part of it. The motivator for this closure has to be some form of authority and here this may be between the different categories of participant. This is also the mode of the scaffolding that goes along with the zone of proximal development and situated cognition, because both are developmental in respect of a closed practice, it being recalled that I am claiming that all practices are cultural arbitraries.

Equilibration is the mode that Piaget hopes for in a pedagogy that is free from authority. Under such conditions, Piaget trusts the student to learn and to arrive, pretty much, at what mathematics predicts. Again though, as a sociologist, I am not going to accept easily the claim that any interaction can truly be between similars and free of authority. It may be asked, why postulate a situation that you deny is possible. But, firstly, claims are made that this is the mode in which pedagogy should take place and so the mode has to be included in the scheme if only to reveal its empirical absence. Secondly, it may be helpful to think in terms of the relative absence of authority or, perhaps, in terms of strategies that might tend to reduce it and so shift between hegemonic and equilibrating modes. But although authority is presumed to have been removed from relations between participants in the interaction, nevertheless, there must be some tension in the situation that motivates the dynamic of equilibration.[10]

The righthand column of Figure 12 again removes authority from the interaction, but here this does not pose the problem identified in the case of equilibration, because there is no drive to closure. The *exchange of narratives* mode consists in a sequence or circle of performances by participants, constituted as similars, similars and so, perhaps, speaking with rather than to or at each other; the probability of change here is minimal. The difference in the case of *pastiche* is that participants are dissimilar. It is in this last form of interaction that the interactants may (or may not) turn in on themselves and respond, in some way, to the alternative with which they have been presented by their interlocutor. Alternatively, they may recruit aspects of the alternative in the elaboration of their own activity. Unlike hegemony and equilibration, however, there is no necessary coordination between the accommodations and assimilations that may or may not take place as we look at individual participants.

Figures 11 and 12 are clearly related and both speak also to the schema in Figure 10. But each asks slightly different questions of the text or of the classroom and this is the point of these and the other schemas in this paper. They do not provide answers to the question, how do we construct a text or a curriculum or a lesson. What they do is provide part of a language to enable the interrogation of texts, curricula and lessons that might, if responses are made earnestly, reveal that things are not always as they seem to be.

Conclusion

The academic field of mathematics education, particularly in relation to primary school mathematics, has a marked tendency to favour general approaches that favour the psychological. The remarkable work of Jean Piaget may have something to do with this (see, for example, Walkerdine, 1984). What I have attempted to do in this paper is to outline an approach that comes from a more sociological perspective and that also foregrounds language, text and context, all of which contrast with cognitive structure in the sense that they can be thought of as directly observable rather than as inferred from linguistic and other action on the basis of theoretical constructs. This is not to say that the kind of approach that I am proposing does not need theory—far from it. But the kind of theory that I want to construct—that I have constructed—is very open about its constructed nature. I now tend to avoid the use of the word ‘theory’ when referring to my own work, in favour of terms such as ‘organisational language’ and ‘method’. So in this paper I have attempted to indicated the basis for this kind of approach,

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[10] I (Dowling, 1996, 1998) have identified this as a flaw in Piaget’s scheme, which is predicated on autoregulation, that is to say, in all senses, the organism must ‘know’ in which direction equilibrium is to be sought.

illustrated how my organisational language arises out of a transaction with empirical texts and contexts, and moved on to the presentation of some schemas that have interrogative value in reflecting upon primary school mathematics texts and contexts. What I have certainly not tried to do is prescribe practices for the construction of texts, curricula or lessons; these practices are not the same kinds of activities as academic research and writing and to recontextualise the former as the public domains of the latter may be interesting, may be inspirational, may provide a critical distance, but it will always involve recontextualisation.

So, whilst the title of the paper is 'What is Mathematics?' I have started out by saying what I think it is not, which is universalistic and inevitable in form. Rather, we should think of mathematics (at any level) as consisting of what Bourdieu and Passeron (1977) referred to as 'cultural arbitraries'. In particular, to the extent that the school privileges particular cultural arbitraries, then we should expect that these will resonate more strongly with some cultural groups than with others. I have then described school mathematics as incorporating an essentially self-referential and primarily DS^+ esoteric domain that is a cultural arbitrary rendered non-arbitrary by the curriculum. There is ample evidence to suggest that such an esoteric domain will be far more consistent with and be far more likely to be recognised by the children of European, professional middle class familes than children having other backgrounds.

I then proposed, in effect, that there is no privileged viewpoint on the world. This is consistent with critical realist philosophy (Bhaskar, 1997, 1998) that asserts, in very general terms, that the patterns that we think we see in our world are the artefacts of the methods that we deploy in observation. These methods are generated in and by the sociocultural groups in which we operate. They vary in terms of DS and in terms of level of institutionalisation: school mathematics tends to be DS^+ and I^+; domestic practice tends to be DS^- and I^-. In Bernstein's (2000) terms, this would associate mathematics with vertical discourse and domestic practice with horizontal discourse; in the terms of Figure 10, these practices would be predominantly discourse and trick, respectively. The organisational language of school mathematics comprises the esoteric domain of mathematical content together with what we can think of as pedagogic theory that enables a gaze to be cast beyond mathematics in the construction of a public domain of recontextualised texts and contexts.

School mathematics makes much of its relationship to the 'real world'. Of course, school is a part of the real world, but the construction of the public domain entails the casting of a gaze onto another part of the real world and transforming it, recontextualising it; public domain domestic practices are not domestic practices. Nevertheless, school mathematics frequently presents the myth that its 'real world' applications constitute a necessary condition for adequate domestic practice and, indeed, for adequacy in a whole range of other non-mathematical activities. I have referred to this myth as the myth of participation (Dowling, 1998; see also Dowling & Brown, 2000): mathematics is a necessary condition for participation in non-mathematical activities. By and large, my contention is that very few people make any use in their out of school lives of any of the mathematics that they learned (or did not learn) in school; they can mostly get on very well without it.

Another construction of the relationship between mathematics and the ‘real world’ establishes the empiricist myth that esoteric domain mathematics emerges naturally from empirical engagement with and in the world. Of course, what we now know as mathematics had its origins in other practices, but it took a very long time indeed to develop and has now taken on a life of its own as a self-referential body of practices. In particular, mathematics is not an empirical activity; you cannot ‘prove’ that the angles of a triangle add up to 180^o by drawing lots of triangles, cutting them out, and placing their angles long a straight edge; you have to work analytically from Euclid’s postulates. There is no natural route into the esoteric domain of mathematics, just as there is no natural route into a natural language, such as English or Greek or Japanese; we have to be taught.

I then introduced the expressive and descriptive domains to complete the domains of practice schema, which is presented in Figure 5. Activities that remain in any one of these domains can serve as traps and lead to students making quite fundamental errors; a fraction is not a piece of cake. Nor, or course, can mathematics education begin and remain exclusively in the esoteric domain; there has to be a way in and this will always be via the public domain. Pedagogic action must then construct trajectories that lead into the esoteric via the expressive and that lead to the public domain from the esoteric via the descriptive. Figure 5 provides a map of the territory; in general, in respect of any specialist region of mathematics, the whole of the map should be traversed in one way or another.

Figure 5 is the first of four schemes that I have presented for the mapping of school mathematics texts and contexts. The second is the scheme in Figure 10, which derives from the contention that mathematical faculties include both DS^+ discourse and DS^- skills that are strongly institutionalised by and in the curriculum. Insofar as mathematics pedagogy claims to encourage the development of idiosyncratic methods and approaches by students, then these two modes of practice are mirrored by the weakly institutionalised forms, idiolect (DS^+) and trick (DS^-). My suspicion is that in actual classroom practice, these weakly institutionalise modes are neutralised in rendering them compatible with their strongly institutionalised counterparts.

Figure 12 presents a modality of authority strategies. As with the schemes in Figure 5 and 11, I would expect any extended pedagogic practice to deploy all four modes. I am though suspicious regarding the extent to which the liberal mode is deployed, though this mode is sometimes regarded as crucial to a constructivist pedagogy. Figure 13 presents a related scheme that is concerned with modes of interaction. Where the target of the interaction is closure, then there must be some kind of motivator for this. In general, we would expect this to derive from authority strategies deployed within the interaction. Piaget, however, trusts the learner to learn in the absence—and to an extent only in the absence—of external authority. The motivation, for Piaget, is provided for in the essential nature of the organism as autoregulating which motivates cognitive equilibration. Certainly, the human body can be described as autoregulative in respect of certain biochemical states. It is not clear, however, why this must transfer to all modes of activity and, indeed, more recent work in the field of cybernetics has tended to move away from homeostasis as a fundamental principle (see Hayles, 1999, for an engaging pastiche of cybernetics and literature). Equilibration in my scheme may be otherwise motivated. The two modes, exchange of narratives and pastiche, however, seem to be predicated on a liberal authority mode, which is to say, they are modes of exchange practice rather than the pedagogic practice that establishes equilibration and hegemony. Insofar as mathematics education lays claims to the value of the liberal mode, then Figures 11 and 12 provide a language for its interrogation in practice.

I have presented elements of my organisational language and attempted to illustrate both the ways in which it has arisen in transaction with empirical texts and contexts and suggested how it might be of value to the primary mathematics educator. More of this language and its deployment in a far wider range of contexts as well as its theoretical and methodological origins are detailed in Dowling (in press). I will, of course, be accused of doing exactly what I accuse school mathematics of doing; mythologising the real world. But I am not. I make no claim to have captured my empirical texts and contexts in producing commentaries upon them. What I have done is to illustrate what the world looks like from a very particular kind of perspective and to offer some resources for further elaborating this picture in parts or in whole; it is entirely up to my audiences and my students as to what to do or not to do with the result; the invitation is for you to recontextualise my text, I hope, to your own advantage.

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